Math 34A, Sep 23, 2024

Review: Secant Lines and Tangent Lines

A **secant line** is a straight line that intersects a curve at two or more distinct points. The slope of the secant line between two points $x = a$ and $x = b$ on the curve $y = f(x)$ is given by the average rate of change of the function over that interval:

$$m_{\text{secant}} = \frac{f(b) - f(a)}{b - a}.$$

The secant line approximates the behavior of the curve over the interval $[a, b]$.

A **tangent line**, on the other hand, is a straight line that touches a curve at exactly one point, without crossing it (locally). It represents the instantaneous rate of change (the derivative) of the function at that point. If the curve is $y = f(x)$ and the tangent line touches the curve at $x = a$, then the slope of the tangent line is:

$$m_{\text{tangent}} = f'(a).$$

The tangent line provides a linear approximation to the curve at the point of tangency.

To find the equation of a secant line or a tangent line, we can consider the point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1),$$

where (x_1, y_1) is one of the points, and m is the slope.

Exercises

Problem 1 Find the equation of the secant line between the points where the function $f(x) = x^2 - 3x + 2$ intersects the curve at $x = 1$ and $x = 3$.

Problem 2 Given the function $f(x) = \sin(x)$, calculate the slope of the secant line between $x = 0$ and $x = \pi/4$.

Problem 3 Determine the equation of the tangent line to the curve $f(x) = 3x^2 - 2x + 1$ at the point where $x = 2$.

Problem 4 Consider the function $f(x) = \ln(x)$. Compute the slope of the secant line between $x = 1$ and $x = e$ and compare it with the slope of the tangent line at $x = e$.

Problem 5 Let $f(x) = e^x$. Find the points on the curve where the slope of the secant line between $x = 0$ and $x = a$ is equal to the slope of the tangent line at $x = a/2$.

Problem 6 Find the equation of the secant line for the function $f(x) = \frac{1}{x}$ between the points $x = 1$ and $x = 2$. Then, calculate the equation of the tangent line at $x = 1.5$.

Problem 7 Consider the function $f(x) = \cos(x)$. Calculate the slope of the secant line between $x = 0$ and $x = \pi/3$. Then, find the slope of the tangent line at $x = \pi/6$.

Problem 8 For the cubic function $f(x) = x^3 - 3x^2 + 2x$, determine where the tangent line is parallel to the secant line passing through the points where $x = 0$ and $x = 2$.

Open question How can we use these methods?